Grade 9

WEEK 9

Lesson # 2

Topic: Algebra

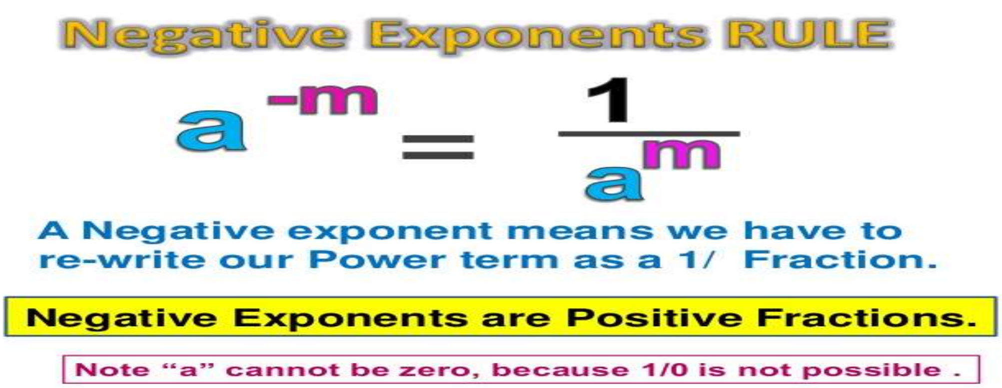
Sub-Topic: Negative and Fractional index

Objectives:

Students will:

- ✓ Break down expressions into index form;
- ✓ Differentiate between fractional and negative index..

Content:



Negative Index

Now $5^3 \div 5^5 = \frac{5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = \frac{1}{5^2}$

And $5^3 \div 5^5 = 5^{3-5} = 5^{-2}$

Thus $5^{-2} = \frac{1}{5^2}$.

Also $q^4 \div q^7 = \frac{q \times q \times q \times q}{q \times q \times q \times q \times q \times q \times q}$

$= \frac{1}{q^3}$

And $q^4 \div q^7 = q^{4-7} = q^{-3}$

Thus $q^{-3} = \frac{1}{q^3}$.

Hence in *general*

$$a^{-m} = \frac{1}{a^m}.$$

That is, a *quantity* with a *negative index* is the *inverse* (or *reciprocal*) of the *quantity* with a *positive index* of the *same magnitude*.

Example 55

(a) Rewrite each of the following expressions using positive index only:

(i) 4^{-3} *(ii)* x^{-5}

(b) Rewrite each of the following expressions using negative index only:

(i) $\frac{1}{3^5}$ *(ii)* $\frac{1}{x^4}$

(b) (i) Now $\frac{1}{3^5} = 3^{-5}$ (*negative* index)

(ii) Now $\frac{1}{x^4} = x^{-4}$ (*negative* index)

(a) Now $25^{\frac{1}{2}} = (5^2)^{\frac{1}{2}} = 5^{2\times\frac{1}{2}} = 5$

Alternatively $25^{\frac{1}{2}} = \sqrt{25} = \sqrt{5^2} = 5$

(b) Now $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{3\times\frac{1}{3}} = 5$

Alternatively $125^{\frac{1}{3}} = \sqrt[3]{125} = \sqrt[3]{5^3} = 5$

Fractional (Rational) Index

Now $4^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^{2\times\frac{1}{2}} = 2^1 = 2$

And $\sqrt{4} = \sqrt{2^2} = 2$

Thus $4^{\frac{1}{2}} = \sqrt{4}$

Hence $4^{\frac{1}{2}}$ is the *square root* of 4.

Also $8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{3\times\frac{1}{3}} = 2^1 = 2$

And $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

Thus $8^{\frac{1}{3}} = \sqrt[3]{8}$

Hence $8^{\frac{1}{3}}$ is the *cube root* of 8.

Now $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{3\times\frac{2}{3}} = 2^2 = 4$

And $\sqrt[3]{8^2} = \sqrt[3]{64} = \sqrt[3]{4^3} = 4$

Thus $8^{\frac{2}{3}} = \sqrt[3]{8^2}$

Hence $8^{\frac{2}{3}}$ is the *cube root* of the *square* of 8.

Now $81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^{4\times\frac{3}{4}} = 3^3 = 27$

And $\sqrt[4]{81^3} = \sqrt[4]{(3^4)^3} = \sqrt[4]{3^{4\times 3}} = 3^3 = 27$

Thus $81^{\frac{3}{4}} = \sqrt[4]{81^3}$

Hence $81^{\frac{3}{4}}$ is the *fourth root* of the *cube* of 81.

NOTE: The *even root of a number* can be either *positive* or *negative*. For example:

$$\sqrt{25} = \pm 5 \text{ and } \sqrt[4]{81} = \pm 3.$$

However we are only taking the *positive root* in this chapter.

Hence in *general* $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

That is, in a *quantity* with a *fractional* (or *rational index*), the *denominator* is the *root* and the *numerator* is the *power* to which the *quantity* is to be *raised*.

Example 56

Determine the value of each of the following:

(a) $25^{\frac{1}{2}}$ *(b)* $125^{\frac{1}{3}}$

(a) Now $25^{\frac{1}{2}} = (5^2)^{\frac{1}{2}} = 5^{2\times\frac{1}{2}} = 5$

Alternatively $25^{\frac{1}{2}} = \sqrt{25} = \sqrt{5^2} = 5$

(b) Now $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{3\times\frac{1}{3}} = 5$

Alternatively $125^{\frac{1}{3}} = \sqrt[3]{125} = \sqrt[3]{5^3} = 5$

Example 57

Determine the value of each of the following:

(a) $32^{\frac{2}{5}}$ *(b)* $27^{\frac{2}{3}}$

Solution

(a) Now $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

Alternatively $32^{\frac{2}{5}} = \sqrt[5]{32^2} = \sqrt[5]{(2^5)^2}$

$= \sqrt[5]{2^{5\times 2}} = 2^2 = 4$

(b) Now $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{3\times\frac{2}{3}} = 3^2 = 9$

Alternatively $27^{\frac{2}{3}} = \sqrt[3]{27^2} = \sqrt[3]{(3^3)^2}$

$= \sqrt[3]{3^{3\times 2}} = 3^2 = 9$

Fractional Indices

Numerator – Power

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

Denominator - Root

Examples:

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$25^{\frac{3}{2}} = \left(\sqrt[2]{25}\right)^3 = 5^3 = 125$

Exercise:

Exercise 6s

Express each of the following products in index form:

1. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
2. $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
3. $5 \times 5 \times 5 \times 5 \times 5 \times 5$
4. $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
5. $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$
6. $x \times x \times x \times x \times x$
7. $y \times y \times y \times y \times y \times y \times y$
8. $a \times a \times a \times \ldots$, to the p^{th} term
9. $m \times m \times m \times \ldots$, to the q^{th} term
10. $z \times z \times z \times \ldots$, to the n^{th} term.

Express each of the following products in index form:

11. $2^5 \times 2^3$
12. $3^4 \times 3^5$
13. $5^6 \times 5^3$
14. $7^3 \times 7^4$
15. $8^5 \times 8^4$
16. $x^2 \times x^4$
17. $y^3 \times y^5$
18. $z^4 \times z^5$
19. $p^7 \times p^3$
20. $q^5 \times q^8$
21. $x^2 \times y^3 \times x^5 \times y^2$
22. $x^4 \times y^2 \times y^3 \times x^7$
23. $p^3 \times q^2 \times p^8 \times q^5$
24. $p^5 \times q^3 \times p^2 \times q^6$
25. $a^3 \times b^2 \times b^3 \times a^4$
26. $p^3 \times q^2 \times r \times p^2 \times q \times r^3$
27. $p^2 \times q^3 \times r^4 \times q^2 \times p \times r$
28. $x^4 \times y^3 \times z^2 \times x^3 \times y^2 \times z$
29. $x^2 \times y^3 \times z^4 \times z^3 \times y^2 \times x^7$
30. $a^5 \times b^2 \times c^3 \times a \times c^2 \times b^3$

Express each of the following quotients in index form:

31. $6^7 \div 6^4$
32. $7^5 \div 7^2$
33. $8^9 \div 8^7$
34. $9^{12} \div 9^7$
35. $10^{13} \div 10^8$
36. $a^9 \div a^5$
37. $x^7 \div x^3$
38. $m^8 \div m^5$
39. $p^{12} \div p^9$
40. $q^{15} \div q^{13}$
41. $x^5y^2 \div x^3y$
42. $x^7y^3 \div x^4y$
43. $p^3q^4 \div pq^3$
44. $r^5s^4 \div r^2s^3$
45. $m^6n^5 \div m^3n^2$
46. $p^7q^5r^3 \div p^4q^2r$
47. $p^8q^7r^4 \div p^5q^2r^3$
48. $x^7y^5z^3 \div x^5y^3z$
49. $l^3m^5n^2 \div lm^3n$
50. $a^5b^4c^3 \div a^2b^3c$

Simplify each of the following expressions ...

Solution:

Exercise 6s

1. 2^7
2. 3^8
3. 5^6
4. 9^7
5. 7^9
6. x^5
7. y^7
8. a^p
9. m^q
10. z^n
11. 2^8
12. 3^9
13. 5^9
14. 7^7
15. 8^9
16. x^6
17. y^8
18. z^9
19. p^{10}
20. q^{13}
21. x^7y^5
22. $x^{11}y^5$
23. $p^{11}q^7$
24. p^7q^9
25. a^7b^5
26. $p^5q^3r^4$
27. $P^3q^5r^5$
28. $x^7y^5z^3$
29. $x^9y^5z^7$
30. $a^6b^5c^5$
31. 6^3
32. 7^3
33. 8^2
34. 9^5
35. 10^5
36. a^4
37. x^4
38. m^3
39. p^3
40. q^2
41. x^2y
42. x^3y^2
43. p^2q
44. r^3s
45. m^3n^3
46. $p^3q^3r^2$
47. p^3q^5r
48. $x^2y^2z^2$

Reference: https://www.bbc.co.uk/bitesize/guides/zpkmpbk/revision/7