Lesson 3

Tuesday, April 2, 2024 6:50 PM

CSEC P2 – 2013 Q1 & Q2.

1. (a) Let $f(x) = x^3 - x^2 - 14x + 24$.

(i) Use the factor theorem to show that $x + 4$ is a factor of $f(x)$. **(2 marks)**

(ii) Determine the other linear factors of $f(x)$. **(3 marks)**

(b) A function $f(x)$ is given by $f(x) = \frac{2x-1}{x+2}$.

(i) Find an expression for the inverse function $f^{-1}(x)$. **(3 marks)**

(ii) The function g is given by $g(x) = x + 1$. Write an expression for the composite function, $fg(x)$. Simplify your answer. **(2 marks)**

(c) Given that $5^{3x-2} = 7^{x+2}$, show that

$x = \frac{2(\log 5 + \log 7)}{(\log 125 - \log 7)}$. **(4 marks)**

Total 14 marks

(b) Functions & Inequalities (Topic)

(i) $f(x) = \frac{2x-1}{x+2}$, $x \neq -2$

$y = \frac{2x-1}{x+2} \Rightarrow x = \frac{2y-1}{y+2}$ ✓

$\Rightarrow x(y+2) = 2y - 1$ ✓

$\Rightarrow xy + 2x = 2y - 1$

$\Rightarrow xy - 2y = -2x - 1$

$\Rightarrow y(x-2) = -2x - 1$

$\Rightarrow y = \frac{-2x-1}{x-2}$ ✓

$\Rightarrow f^{-1}(x) = \frac{-2x-1}{x-2}$ (Answer)

(ii) $g(x) = x + 1$, Find: $f[g(x)]$: ?

$x = g(x)$

$\Rightarrow x = x + 1$

$\Rightarrow f(x) = \frac{2(x+1) - 1}{(x+1) + 2}$

$= \frac{2x + 2 - 1}{x + 1 + 2}$

$fg(x) = \frac{2x+1}{x+3}$ (Answer).

(c) $5^{3x-2} = 7^{x+2}$ ✓ $\quad x = \frac{2(\log 5 + \log 7)}{}$

(c) $5^{3x-2} = 7^{x+2}$ ✓ $\left[x = \frac{2(\log 5 + \log 7)}{\log 125 - \log 7}\right]$

Take logs on both sides: ✗

$\log_{10} 5^{3x-2} = \log_{10} 7^{x+2}$

ln lg

$(3x-2)\log 5 = (x+2)\log 7$

$3x\log 5 - 2\log 5 = x\log 7 + 2\log 7$

$\Rightarrow 3x\log 5 - x\log 7 = 2\log 5 + 2\log 7$

$\Rightarrow x(3\log 5 - \log 7) = 2(\log 5 + \log 7)$

$\Rightarrow x(\log 5^3 - \log 7) = 2(\log 5 + \log 7)$

$\Rightarrow x = \frac{2(\log 5 + \log 7)}{\log 125 - \log 7}$

2. (a) Let $f(x) = 3x^2 + 6x - 1$.

(i) Express $f(x)$ in the form $a(x + h)^2 + k$ where h and k are constants. **(3 marks)**

(ii) State the minimum value of $f(x)$. **(1 mark)**

(iii) Determine the value of x for which $f(x)$ is a minimum. **(1 mark)**

(b) Find the set of values of x for which $2x^2 + 3x - 5 \geq 0$. **(4 marks)**

(c) Find the sum to infinity of the following series:

$\frac{1}{4} + \frac{2}{4^2} + \frac{1}{4^3} + \frac{2}{4^4} + \ldots$

Note: This series can be rewritten as the sum of two geometric series. **(5 marks)**

Total 14 marks

(a)(i) $f(x) = 3x^2 + 6x - 1 \Rightarrow a = 3$

$\Rightarrow h = \frac{b}{2a} = \frac{6}{2(3)} = 1$

$\Rightarrow k = \frac{4ac - b^2}{4a} = \frac{4(3)(-1) - (6)^2}{4(3)}$

$= \frac{-12 - 36}{12} = -\frac{48}{12} = -4$

$a(x+h)^2 + k = 3(x+1)^2 - 4$

(ii) $f(x)_{min.} = -4$

$(x+1) = 0$

(iii) $x_{min.} = -1$

$(x+1) - c$

(ii) $x_{min.} = -1$

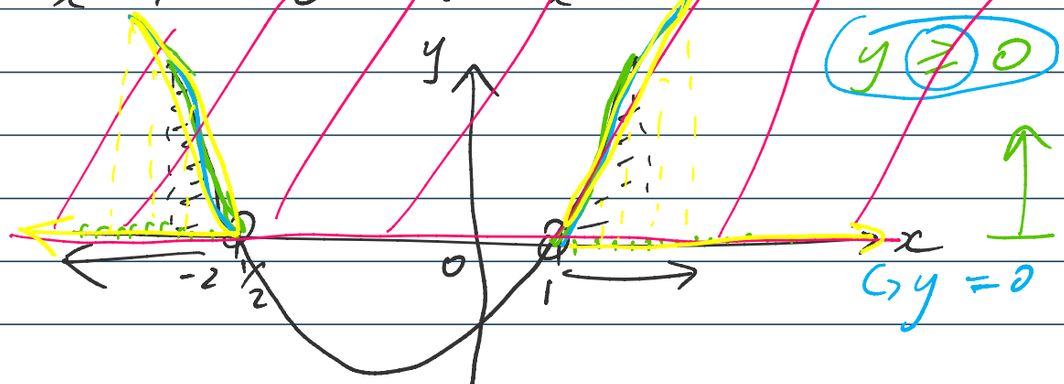
(b) $2x^2 + 3x - 5 \geq 0$ → $f(x)$ / y

$a > 0$ ∴ min. curve.

$(2x+5)(x-1) \geq 0$

$\Rightarrow 2x + 5 = 0 \quad \therefore x = -5/2 = -2\frac{1}{2}$

$\Rightarrow x - 1 = 0 \quad \therefore x = 1$



$\{x : x \leq -2\frac{1}{2}\} \cup \{x : x \geq 1\}$ ✓